

**GCE**

**Further Mathematics A**

**Y531/01: Pure Core**

Advanced Subsidiary GCE

**Mark Scheme for June 2019**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## Text Instructions

## Annotations and abbreviations

| <b>Annotation in scoris</b>               | <b>Meaning</b>   |
|---|--|
| ✓and*                                     |  |
| BOD                                       | Benefit of doubt   |
| FT  | Follow through   |
| ISW                                       | Ignore subsequent working  |
| M0, M1                                    | Method mark awarded 0, 1   |
| A0, A1                                    | Accuracy mark awarded 0, 1   |
| B0, B1                                    | Independent mark awarded 0, 1  |
| SC  | Special case   |
| ^   | Omission sign  |
| MR  | Misread  |
| Highlighting                              |  |
|   |  |
| <b>Other abbreviations in mark scheme</b> | <b>Meaning</b>   |
| E1  | Mark for explaining a result or establishing a given result                                |
| dep*                                      | Mark dependent on a previous mark, indicated by *  |
| cao                                       | Correct answer only  |
| oe  | Or equivalent  |
| rot                                       | Rounded or truncated   |
| soi                                       | Seen or implied  |
| www                                       | Without wrong working  |
| AG  | Answer given   |
| awrt                                      | Anything which rounds to   |
| BC  | By Calculator  |
| DR  | This question included the instruction: In this question you must show detailed reasoning. |

**Subject-specific Marking Instructions for AS Level Further Mathematics A**

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep\*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
  - When a value is not given in the paper accept any answer that agrees with the correct value to **3 s.f.** unless the question specifically asks for another level of accuracy.
- NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".  
Follow through should be used so that only one mark is lost for each distinct accuracy error.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question |     | Answer  | Marks   | AO                        | Guidance   |
|----------|-----|---|---|---------------------------|--|
| 1        | (a) | $ z  = 5$<br>$\arg z = -0.927$ rads or $-53.1^\circ$<br><br>$z^* = 3 + 4i$  | <b>B1</b><br><br><b>B1</b><br><br><b>B1</b><br><b>[3]</b> | 1.1<br><br>1.1<br><br>1.1 | From $\sqrt{3^2 + 4^2}$ or <b>BC</b><br>From $\tan^{-1}(-\frac{4}{3})$ , $\tan \theta = \frac{4}{3}$ or <b>BC</b><br><br>Or 5.36 rads (5.35589...) or $307^\circ$ (or 306.8698...)   |
|          | (b) | A and B are reflections of each other...<br><br>... in the real (or horizontal or x) axis   | <b>M1</b><br><br><b>A1</b><br><br><b>[2]</b>              | 1.1<br><br>1.1            | Reflection / Reflected<br><br>Correct mirror line<br>$y = 0$ ok<br>Do not allow "positive real axis"<br><br>Allow references to w and w* (or z and z* etc) rather than A and B. Do not allow "mirrored" unless also a reference to "reflection"<br><br>Could describe the geometrical relationship in terms not involving the word "reflection" but would need to be entirely correct and un-ambiguous.<br><br>Diagram only is no marks. Diagram with accompanying description for a general case is fine. |
| 2        | (a) | B<br>(For matrices to be conformable for multiplication) the number of columns of the first must equal the number of rows in the second oe<br>"the number of rows of P is equal to the number of columns of Q"                | <b>B1</b><br><b>E1</b><br><br><b>[2]</b>                  | 2.2a<br>1.2               | Note "B" is that QP is conformable<br>Statement can be general or specific.<br>Allow eg $(1 \times 2) \times (2 \times 3) = (1 \times 3)$<br>provided that it is clear which two numbers must be the same<br><br>Since told exactly one is true it is sufficient to give a reason why one is true or why one is false  |
|          | (b) | $\mathbf{QP} = \begin{pmatrix} 1+k & -1 \end{pmatrix} \begin{pmatrix} 1 & k & 0 \\ -2 & 1 & 3 \end{pmatrix}$ $= \begin{pmatrix} (1+k)+2 & k(1+k)-1 & -3 \end{pmatrix}$ $\begin{pmatrix} (k+3) & (k^2+k-1) & -3 \end{pmatrix}$ | <b>M1</b><br><br><b>A1</b><br><br><b>[2]</b>              | 1.1<br><br>1.1            | Correct method for multiplying matrices (can be implied by any one entry correct)<br><br>Accept un-simplified elements<br><br>If <b>PQ</b> attempted then <b>M0A0</b> unless explicitly rejected   |



| Question | Answer   | Marks   | AO                               | Guidance   |   |
|----------|--|---|----------------------------------|--|---|
| (e)      | Need $9\mathbf{i} + 8\mathbf{j} - 12\mathbf{k} = p\mathbf{a} + \mu\mathbf{b}$<br><br>(any two of) $-9p - 2\mu = 9$ , $2p + \mu = 8$ ,<br>$6p + \mu = -12$<br>$p = -5$            | <b>M1</b><br><b>M1</b><br><b>A1</b><br><b>[3]</b>                           | 2.2a<br>1.1<br>1.1               | Could instead consider<br>$p\mathbf{a} = 9\mathbf{i} + 8\mathbf{j} - 12\mathbf{k} + \mu\mathbf{b}$<br>Equating coefficients for two of <b>i</b> , <b>j</b> and <b>k</b><br><br>BC or eliminating $\mu (=18)$   | This would give $\mu = -18$<br><br>Only 2 equations are necessary<br>$\mu$ might be the negative value<br>No need to check for consistency  |
| 4 (a)    | $2 - i$ (is also a root)<br>$(z - (2 + i))(z - (2 - i))$<br>$(z - 2)^2 + 1$<br>$z^2 - 4z + 5$<br><br>$(z^2 - 4z + 5)(4z^2 + \dots + 37)$<br><br>$(z^2 - 4z + 5)(4z^2 + 4z + 37)$ | <b>B1</b><br><b>M1</b><br><b>A1</b><br><b>M1</b><br><b>A1</b><br><b>[5]</b> | 2.2a<br>1.1<br>2.1<br>1.1<br>1.1 | Can be implied by $(z - (2 - i))$<br>Correctly determining the form of the quadratic in factor form<br>Working needs to be shown here (Show detailed reasoning). At least 1 step between factor form and final answer<br>With non-zero $z$ term. Or attempt at symbolic division resulting in at least $4z^2$ .<br>Or genuine attempt at method comparing coefficients leading to at least one useful equation (eg $z^4: A = 4$ )<br>Must be a product of two quadratics | DR: Detailed reasoning required for this question<br><br>Multiplication table might be used (backwards)<br>Inspection is fine here, so candidates might just write down the correct two quadratic factors.  |
| (b)      | $\frac{-4 \pm \sqrt{4^2 - 4 \times 4 \times 37}}{2 \times 4}$ soi<br>$\frac{-4 \pm 24i}{8}$ or $\frac{-1 \pm 6i}{2}$ etc.<br>$\frac{-1 \pm 6i}{2}$ , $2 \pm i$                   | <b>M1*</b><br><b>A1dep(*)</b><br><b>A1(ft) dep(*)</b><br><b>[3]</b>         | 1.1a<br>2.1<br>1.1               | Correct method for finding a root of a quadratic ( $\pm$ not necessary)<br>Simplification of the square root to include $i$ ( $\pm$ is now necessary)<br><br>All four - aef<br>Follow through on incorrect roots from previous mark as long as method mark awarded i.e. this mark is for realising that there are four roots and including the two from part (a). <b>A0</b> if more than 4 roots given.  | or $4((z + 1/2)^2 - 1/4) + 37$ oe<br>Need to see some method<br><br>Must be of the form $z = a + bi$ (or $z = \frac{a+bi}{c}$ ) No square roots left in.<br><br><b>SC</b> : If <b>M0</b> awarded but all four correct roots are given with no extra then award <b>SC B1</b> |



| Question | Answer  | Marks  | AO                           | Guidance  |
|----------|---|--|------------------------------|---|
| (c)      | $r_2 =  2 \pm i  = \sqrt{5}$ $r_1 = \left  \frac{-1 \pm 6i}{2} \right  = \frac{\sqrt{37}}{2}$ $\pi \left( \frac{\sqrt{37}}{2} \right)^2 - \pi (\sqrt{5})^2$ <p>The area between the two circles is <math>\frac{17}{4}\pi</math></p> | <b>B1</b><br><b>B1ft</b><br><b>M1</b><br><b>A1ft</b><br><b>[4]</b> | 2.2a<br>2.2a<br>3.1a<br>3.2a | Need to see $ 2+i $ or $ 2-i $<br>ft the magnitude of the other of their conjugate pairs<br>Same comment as above.<br>Either way round. Their $r_1$ and/or $r_2$ .<br><br>$\pi(r_1^2 - 5)$ simplified<br>www<br><br>Or $\sqrt{2^2 + 1^2} = \sqrt{5}$ etc.<br><br>Positive area needed here. Must follow <b>B1B1(ft)M1</b>   |
| (d)      | $\omega = -\frac{-12}{4} = 3\dots$ <p>...and <math>\sqrt{5} &lt; 3 &lt; \frac{\sqrt{37}}{2}</math> so <math>\omega</math> is in <math>R</math></p>  | <b>B1</b><br><b>E1</b><br><br><b>[2]</b>                           | 2.2a<br>2.3                  | or $\frac{-1+6i}{2} + \frac{-1-6i}{2} + 2+i+2-i = 3$<br>Or $5 < 9 < 37/4$<br>Or $2.23\dots < 3 < 3.04\dots$<br>Both end comparisons needed for E1<br>No follow through given in this part<br><br>Some justification needed, not just $\omega = 3$ . Allow $3+0i$<br>Conclusion needed<br>Can be smaller radius is 2.24, larger radius is 3.04 so $\omega$ is in $R$<br>Diagram is ok for <b>E1</b> if it implies both end comparisons |

| Question |     | Answer   | Marks   | AO   | Guidance  |  |
|----------|-----|--|---|--|---|--|
| 5        | (a) | $\alpha\beta\gamma = -\frac{1}{5}, \sum \alpha\beta = \frac{3}{5}, \sum \alpha = \frac{2}{5}$ $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 = \sum \alpha^2 \beta^2 + \sum 2\alpha\beta\gamma\alpha$ $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 = \sum \alpha^2 \beta^2 + 2\alpha\beta\gamma \sum \alpha$ $\sum \alpha^2 \beta^2 = \frac{13}{25}$ | <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>            | <p>1.1a</p> <p>1.1</p> <p>3.1a</p> <p>1.1</p> <p>1.1</p> | <p>Any one of <math>\alpha\beta\gamma, \alpha\beta + \beta\gamma + \gamma\alpha, \alpha + \beta + \gamma</math></p> <p>All three correct</p> <p>Expansion showing or implying 6 terms including squares and cross-terms, symmetrical in <math>\alpha, \beta</math> and <math>\gamma</math>. Might have missing 2</p> <p>Correct, useful form</p>  | <p>DR: Detailed reasoning required for this question</p> <p><b>See appendix for Special Cases for this question.</b></p> <p>Could see early substitution (see alternative scheme)</p>  |
|          | (a) | <p><b>Alternate</b></p> <p>Substitution <math>x = \sqrt{u}</math> or <math>x^2 = u</math> used</p> $5u\sqrt{u} - 2u + 3\sqrt{u} + 1 = 0$ $((5u + 3)\sqrt{u})^2 = (2u - 1)^2$ $25x^3 + 26x^2 + 13x - 1 = 0$ $\sum \alpha^2 \beta^2 = \frac{13}{25}$   | <p><b>B1</b></p> <p><b>M1*</b></p> <p><b>M1*</b></p> <p><b>A1</b></p> <p><b>A1ft dep(*)</b></p> |  | <p>Soi by correct substitution – must be used</p> <p>Substitution and dealing with <math>(\sqrt{u})^3</math> term</p> <p>Rearrangement with <math>\sqrt{u}</math> terms collected on one side and squaring. LHS does not need to be factorised before squaring</p> <p>Correct expansion and rearranging. Could be in terms of “u”. Or any non-zero integer multiple. Must be an equation.</p> <p>Correct coefficient ratio identified. Must gain method marks and have a cubic.</p> | <p><b>See appendix for Special Cases for this question.</b></p> <p>Must see the correct entire cubic for this mark if using this method.</p> <p>Cubic might not be completely correct but coefficients of <math>x^3</math> and <math>x</math> must be.</p> |
|          |     |  | [5]   |  |   |  |

| Question |     | Answer   | Marks  | AO   | Guidance   |   |
|----------|-----|--|--|--|--|---|
|          | (b) | $\alpha^2 \beta^2 \gamma^2 = (\alpha\beta\gamma)^2 = \frac{1}{25}$ $(\alpha + \beta + \gamma)^2 = \sum \alpha^2 + 2\sum \alpha\beta$ $\sum \alpha^2 = -\frac{26}{25}$ $25x^3 + 26x^2 + 13x - 1 = 0$  | <p><b>B1ft</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>  | <p>2.2a</p> <p>3.1a</p> <p>1.1</p> <p>1.1</p>            | <p>Expansion showing or implying 6 terms including squares and cross-terms, symmetrical in <math>\alpha, \beta</math> and <math>\gamma</math>. Might have 2 missing.</p> <p>Seen or implied</p> <p>Or any non-zero integer multiple. Must be = 0. <b>CAO</b></p>   | <p><b>See appendix for Special Cases for this question.</b></p> $\sum \alpha^2 = \left(\frac{2}{5}\right)^2 - 2 \times \frac{3}{5}$ |
|          | (b) | <p><b>Alternate</b></p> <p>Substitution <math>x = \sqrt{u}</math> or <math>x^2 = u</math> used</p> $5u\sqrt{u} - 2u + 3\sqrt{u} + 1 = 0$ $\left((5u + 3)\sqrt{u}\right)^2 = (2u - 1)^2$ $25x^3 + 26x^2 + 13x - 1 = 0$  | <p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>  |  | <p>Soi by correct substitution - must be used</p> <p>Substitution and dealing with <math>(\sqrt{u})^3</math> term</p> <p>Rearrangement with <math>\sqrt{u}</math> terms collected on one side and squaring. LHS does not need to be factorised before squaring</p> <p>Correct expansion and rearranging. Could be in terms of “u”. Or any non-zero integer multiple. Must be = 0. <b>CAO</b></p> | <p><b>See appendix for Special Cases for this question.</b></p>   |
|          |     |  | <b>[4]</b>   |  |  |   |
| 6        |     | $(\Delta =) \begin{vmatrix} x^2 + 1 & -4 \\ 3 - 2x^2 & x^2 + 5 \end{vmatrix}$ $= (x^2 + 1)(x^2 + 5) - (-4)(3 - 2x^2)$ $= x^4 - 2x^2 + 17$ $\Delta = (x^2 - 1)^2 + 16$ <p><math>(x^2 = 1) \Rightarrow \Delta_{\min} = 16</math></p> <p>So the smallest possible area is 192 (units)</p> | <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1*</b></p> <p><b>A1</b></p> <p><b>A1ft (dep*)</b></p> <p><b>[5]</b></p> | <p>1.1</p> <p>1.1</p> <p>3.1a</p> <p>1.1</p> <p>3.2a</p> | <p>Expanding the determinant of <b>T</b>. Condone sign error and/or one “x” rather than “x<sup>2</sup>”</p> <p>Attempt to find minimum value of quadratic in <math>x^2</math></p> <p>Their <math>\Delta_{\min} \times 12</math></p>  | <p>or from <math>\frac{d\Delta}{dx} = 4x^3 - 4x = 0</math></p>  |

| Question |     | Answer   | Marks  | AO                                 | Guidance  |
|----------|-----|--|--|------------------------------------|---|
| 7        | (a) | $x = 5 \Rightarrow \det \mathbf{A} = 2 \times 5^3 - 4 \times 5^2 - 58 \times 5 + 60$<br>$= 250 - 100 - 290 + 60 = -80$<br><br>So vol of $H^1 = 6 \times 80 = 480$ cao<br>...and $\mathbf{A}$ does not preserve the orientation because $\det \mathbf{A} < 0$ . | <b>B1</b><br><br><br><br><b>B1</b><br><b>E1 ft</b><br><br><b>[3]</b>   | 1.1<br><br><br><br>1.1<br>2.4      | Could see e.g.-<br>$\begin{vmatrix} -1 & 5 & 2 \\ 2 & -6 & 1 \\ 5 & -25 & 10 \end{vmatrix}$ $= -1(-60 + 25)$<br>$-5(20 - 5) + 2(-50 + 30)$<br>$= 35 - 75 - 40 = -80$<br><br>If positive then “ $\mathbf{A}$ does preserve the orientation because $\det \mathbf{A} > 0$ ” |
| 7        | (b) | Image coplanar $\Rightarrow \det \mathbf{A} = 0$ soi<br>$\det \mathbf{A} = -1(-6 \times 2x + 5x) - x(2x(7 - x) - 5)$<br>$+ 2(-5x(7 - x) + 6 \times 5)$<br>$2x^3 - 4x^2 - 58x + 60$<br>1, 6, -5   | <b>B1</b><br><b>M1</b><br><br><br><b>A1</b><br><b>A1</b><br><b>[4]</b> | 2.2a<br>3.1a<br><br><br>1.1<br>1.1 | Attempt to expand determinant<br><br><br>Don't need “=0” here<br><b>BC.</b> All three<br><br>No working required for roots  |

| Question | Answer  | Marks   | AO  | Guidance   |
|----------|---|---|---|--|
| 8        | $\mathbf{M}^1 = \begin{pmatrix} 1 & 3(2^{1+1} - 2) \\ 0 & 2^1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \times 2 \\ 0 & 2 \end{pmatrix}$ $= \begin{pmatrix} 1 & 6 \\ 0 & 2 \end{pmatrix} = \mathbf{M}$ <p>Assume true for <math>n = k</math> ie <math>\mathbf{M}^k = \begin{pmatrix} 1 &amp; 3(2^{k+1} - 2) \\ 0 &amp; 2^k \end{pmatrix}</math>.</p> $\mathbf{M}^{k+1} = \mathbf{M}\mathbf{M}^k = \begin{pmatrix} 1 & 6 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 3(2^{k+1} - 2) \\ 0 & 2^k \end{pmatrix}$ $\begin{pmatrix} 1 & 3(2^{k+1} - 2) + 6 \times 2^k \\ 0 & 2 \times 2^k \end{pmatrix}$ $\begin{pmatrix} 1 & 3(2^{k+1} - 2) + 3 \times 2^{k+1} \\ 0 & 2 \times 2^k \end{pmatrix} = \begin{pmatrix} 1 & 3(2^{(k+1)+1} - 2) \\ 0 & 2^{(k+1)} \end{pmatrix}$ <p>So true for <math>n = k \Rightarrow</math> true for <math>n = k + 1</math>. But true for <math>n = 1</math>.<br/>So true for all positive integer <math>n</math></p> | <p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b><br/><b>(AG)</b></p> <p><b>E1</b></p> <p><b>[6]</b></p> | <p>3.1a</p> <p>2.1</p> <p>1.1</p> <p>1.1</p> <p>2.2a</p> <p>2.4</p> | <p>Full details must be shown.</p> <p>Must have statement in terms of some other variable than <math>n</math>.</p> <p>Uses inductive hypothesis properly</p> <p>Genuine attempt at matrix multiplication (ie columns multiplied into rows)</p> <p>Simplification with sufficient working to establish truth for <math>k + 1</math></p> <p>Clear conclusion for induction process. Needs a <b>fully correct proof</b> which usually means all other marks awarded. Cannot be awarded if there are mistakes in the proof.</p> <p>DR: Detailed reasoning required for this question</p> <p>Watch out for <math>2^{k+1}</math> appearing prematurely as bottom right element<br/>or</p> $\mathbf{M}^k \mathbf{M} = \begin{pmatrix} 1 & 3(2^{k+1} - 2) \\ 0 & 2^k \end{pmatrix} \begin{pmatrix} 1 & 6 \\ 0 & 2 \end{pmatrix}$ $\begin{pmatrix} 1 & 6 + 2 \times 3(2^{k+1} - 2) \\ 0 & 2^k \times 2 \end{pmatrix}$ <p>Must see at least one stage of working between expression for <math>\mathbf{M}^k \mathbf{M}</math> (or <math>\mathbf{M}\mathbf{M}^k</math>) and required expression for <math>\mathbf{M}^{k+1}</math> (e.g.:</p> $\begin{pmatrix} 1 & 3(2 + 2 \times 2^{k+1} - 4) \\ 0 & 2^k \times 2 \end{pmatrix}$ $= \begin{pmatrix} 1 & 3(2^{(k+2)} - 2) \\ 0 & 2^{(k+1)} \end{pmatrix}$ <p>A formal proof by induction is required for full marks.</p> <p><b>SC</b> If <b>B1M1M1M1</b> gained, but <b>A0</b> because of lack of intermediate step of working then allow <b>SC B1</b> for fully correct ending statement</p> |

**Appendix: Special cases for question 5 using the substitution method**

**Working is shown in part (a). If working shown in part (b) mark as mark scheme above. If little or no working then mark part (b) as:**

1. Working in (a) is **fully correct**. Then award:
  - 4/4 If correct cubic equation is re-written and there is a reference to the working in part (a) such as “See working above” or arrows etc.
  - 3/4 If correct cubic equation re-written and there is no reference to the working in part (a)
  - 0/4 If an incorrect cubic is written down, or “=0” missing
  
2. Working in (a) is **not** fully correct. Then award:
  - 0/4 if B1 M1 M1 is not awarded in part (a) (i.e. at least three marks given for (a))
  - 3/4 if at least three marks awarded in part (a) and reference is made to the working in part (a) and the same cubic as in part (a) is re-written in part (b) with and “=0”
  - 2/4 if at least three marks awarded in part (a) and the same cubic as in part (a) is re-written in part (b) with and “=0” but **no reference** is made to the working in part (a)

**Working is shown in part (b) and no working shown in part (a).**

1. Working in part (b) is fully correct. Then award:
  - 5/5 if a reference to the working in part (b) is made, the correct cubic **equation** is written down, there is a comment stating that the roots of the new cubic are  $\alpha^2, \beta^2, \gamma^2$  and the correct answer is given.
  - 4/5 as above but no comment about the roots of the new cubic **equation** being  $\alpha^2, \beta^2, \gamma^2$  or there is no reference to the working in part (b) (i.e. one element of explanation is missing).
  - 3/5 if the correct cubic **equation** is written down and correct answer of  $\frac{13}{25}$
  - 3/5 if a reference is made to working in part (b) but the cubic equation is not re-written and no comment about the roots being  $\alpha^2, \beta^2, \gamma^2$  (and correct answer of  $\frac{13}{25}$  is given)

2. Working in part **(b)** is not fully correct.
- 3/5 if a reference to the working in part **(b)** is made, the same cubic **equation** from part **(b)** is written down, there is a comment stating that the roots of the new cubic are  $\alpha^2, \beta^2, \gamma^2$  and the correct follow through coefficient ratio is given.
  - 2/5 as above but no comment about the roots of the new cubic **equation** being  $\alpha^2, \beta^2, \gamma^2$  or there is no reference to the working in part **(b)** (i.e. one element of explanation is missing).
  - 1/5 if same cubic equation as in part **(b)** is written down and the correct follow through coefficient ratio is given.
3. If  $\frac{13}{25}$  appears as an answer in part **(a)** with no supporting working or comments then 0/5

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